Formal Language Foundations and Schema Languages

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Overview

1. XML Languages and Grammars
   - Introduction and Basics
   - Characterization

2. One-Unambiguous Regular Languages
   - Introduction and Basics
   - Recognition
   - Closure

3. Analysis of XML Schema Languages
   - Introduction and Basics
   - Language Classes
   - Evaluating XML Schema Languages
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XML:
- general-purpose markup language widely in use,
- syntactic structure described by XML schema languages.
  - Schema languages (like DTD) define the relative positions of pairs of corresponding tags.

What we do now:
- characterize the language class generated by DTDs,
  - What can we do with XML languages generated by a DTD?
  - What can we not do?
- transform (rather naively) DTDs to string grammars,
- analyze the languages created by these grammars.
  - How can we determine if a given language is in this language class?
Motivation

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- general-purpose markup language widely in use,
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- analyze the languages created by these grammars.
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Definition of XMLGrammars

A is the set of opening tags, \( \overline{A} \) is the set of closing tags, \( r_a \) is a regular expression for each tag sort \( a \).

**Definition: XML Grammars**

Grammar \( G = (N, T, S, P) \) with:

- \( N = X_a \) for all \( a \in A \),
- \( T = A \cup \overline{A} \),
- some \( S \in N \),
- \( P = \{X_a \rightarrow ar_a\overline{a}\} \) with \( a \in A \), \( \overline{a} \in \overline{A} \), \( X_a \in N \).

Constraint: No empty tags and attributes, only reduced grammars.

**Note**

XML grammars as defined above only cover XML languages generated by DTDs.
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Examples of XML Grammars and Dyck Primes

Example

\{a^n\overline{a}^n\} is an XML language generated by \(X \rightarrow a(X|\varepsilon)\overline{a}\).

Definition

The language \(D_A\) (or just \(D\)) of Dyck primes over \(T = A \cup \overline{A}\) is generated by:

\[
X \rightarrow \sum_{a \in A} X_a \\
X_a \rightarrow aX^*\overline{a}, \text{ for } a \in A
\]

\(D_A\) is the language of properly tag-parenthesized words. \(D_A\) is not an XML language (but \(bD_Ab\) is).

\(D_a\) (\(a \in A\)) is the subset of \(D_A\), where each word starts with \(a\) and ends with \(\overline{a}\). \(D_a\) is an XML language.
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**Definition**

$L_G(X)$ is the language generated by a grammar $G$ if $X$ has been chosen as start symbol.

Hence $L_G(X)$ is the set of all words that can be generated from the non-terminal symbol $X$ in the grammar $G$.

**Definition: Contexts in $L$ of Word $w$**

$C_L(w)$ is the set of pairs of words $(x, y)$ such that $xwy \in L$.

**Example:** $L = \{abc^n \mid n \in \mathbb{N}\}$

$C_L(b) = \{(a, c^n) \mid n \in \mathbb{N}\}$
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At first we need to introduce the following definitions.

**Definition**

\[ F_a(L) := D_a \cap F(L) \text{ for each } a \in A, \text{ where } F(L) \text{ is the set of factors of } L. \]

**Example:** \( L = \{a(b\overline{b})^{n}(c\overline{c})^{n}\overline{a} \mid n \geq 1\} \)

\[ F_a(L) = L, \quad F_b(L) = \{b\overline{b}\}, \quad F_c(L) = \{c\overline{c}\}. \]

**Definition**

If \( w \) is a Dyck prime in \( D_a \) it can be uniquely factorized as \( au_{a_1}u_{a_2} \cdots u_{a_n}\overline{a} \) with \( u_{a_i} \in D_{a_i} \) for \( i = 1, \ldots, n \). Then \( a_1a_2 \cdots a_n \in A^* \) is what is called the **trace** of the word \( w \).
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Conditions for a Language to Be XML 2/4

Example

$bd$ is the trace of $abc\overline{c}bd\overline{d}a$, $c$ is the trace of $bccb$.

Definition: Surface

$S_a(L) = \text{set of all traces of words in } F_a(L)$.

Example: $L = \{a(bb)^n(c\overline{c})^n\overline{a} | n \geq 1\}$

- $S_a(L) = \{b^n c^n | n \geq 1\}$
- $S_b(L) = S_c(L) = \{\varepsilon\}$
Conditions for a Language to Be XML 2/4

Example

\[ bd \] is the trace of \( abc \bar{c} b \bar{d} \bar{d} a \),
\[ c \] is the trace of \( bcc \bar{b} \).

Definition: Surface

\[ S_a(L) = \text{set of all traces of words in } F_a(L). \]

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**Example**

bd is the trace of abc\textoverline{a}bd\textoverline{a},
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### Conditions for a Language to Be XML 3/4

**Definition**

1. \( \emptyset \) (the empty set) is a **regular set**.
2. \( \{ \varepsilon \} \) is a regular set.
3. Every finite set is a regular set.
4. If \( R \) and \( S \) are regular sets, then \( R \cup S \), \( RS \), and \( R^* \) also are.

**Theorem**

A language \( L \) over \( A \cup \overline{A} \) is an XML language if and only if the following three conditions hold true:

1. \( L \subseteq D_\alpha \) for some \( \alpha \in A \),
2. \( C_L(w) = C_L(w') \) for all \( a \in A \) and \( w, w' \in F_a(L) \),
3. \( S_a(L) \) is a regular set for all \( a \in A \).
Conditions for a Language to Be XML 3/4

**Definition**

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Conditions for a Language to Be XML 4/4

Example

Non-XML grammar with start symbol $S$:

$$
S \rightarrow aTT\overline{a} \\
T \rightarrow aTT\overline{a} \mid bb
$$

- $L \subseteq D_a$ and $F_a(L) = L$,
- all $w \in L$ share the same $C_L(w)$ (by construction),
- $S_a(L) = (a \cup b)^2$ and $S_b(L) = \{\varepsilon\}$, i.e. both surfaces are regular.

All three conditions are satisfied. $\Rightarrow$ This grammar describes an XML language. $\Rightarrow$ There must be an XML grammar generating this language:

$$
S \rightarrow a(S \mid T)(S \mid T)\overline{a} \\
T \rightarrow bb
$$
Conditions for a Language to Be XML 4/4

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S \rightarrow a(S \mid T)(S \mid T)\overline{a} \\
T \rightarrow b\overline{b}
$$
XML languages are closed neither under union nor difference.

Proof by counter-example:

- Consider $L = D_{\{a,b\}}^*$, $M = D_{\{a,d\}}^*$, and $H = \{cLc\} \cup \{cMc\}$,
- $\{cLc\}$ and $\{cMc\}$ both are XML languages,
- $cab\overline{bac}$ and $ca\overline{add}dc$ are in $H$,
- $(c,d\overline{dc})$ is in $C_H(ab\overline{a})$, so it also has to be in $C_H(ab\overline{ba})$,
- but $cab\overline{bad}\overline{dc}$ is not in $H \Rightarrow$ XML languages are not closed under union,
- then (as direct consequence of De Morgan’s theorem) XML languages are not closed under difference either.
Theorem

XML languages are closed neither under union nor difference.

Proof by counter-example:

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- $cabbac$ and $ca\overline{d}d\overline{c}$ are in $H$,
- $(c, d\overline{d}c)$ is in $C_H(a\overline{a})$, so it also has to be in $C_H(aba\overline{b}a)$,
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Closure Under Union and Difference

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XML Languages and Grammars
One-Unambiguous Regular Languages
Analysis of XML Schema Languages

Closure Under Union and Difference

Theorem

**XML languages are closed neither under union nor difference.**

Proof by counter-example:

- Consider $L = D^*_{\{a,b\}}$, $M = D^*_{\{a,d\}}$, and $H = \{cL\bar{c}\} \cup \{cM\bar{c}\}$,
- $\{cL\bar{c}\}$ and $\{cM\bar{c}\}$ both are XML languages,
- $cabbac$ and $ca\bar{d}\bar{d}d$ are in $H$,
- $(c, d\bar{d}c)$ is in $C_H(a\bar{a})$, so it also has to be in $C_H(abba)$,
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Proof by counter-example:

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XML languages are closed neither under union nor difference.

Proof by counter-example:

- Consider $L = D^{\{a,b\}}$, $M = D^{\{a,d\}}$, and $H = \{cL \overline{c}\} \cup \{cM \overline{c}\}$,
- $\{cL \overline{c}\}$ and $\{cM \overline{c}\}$ both are XML languages,
- $cab\overline{bac}$ and $ca\overline{add}dc$ are in $H$,
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- but $cab\overline{baddc}$ is not in $H$ $\Rightarrow$ XML languages are not closed under union,
- then (as direct consequence of De Morgan’s theorem) XML languages are not closed under difference either.
More Results

- XML languages are closed under intersection.
- For each XML language $L$ there is exactly one reduced XML grammar generating $L$ if variable names and entities are ignored.
- It is decidable if an XML language $L$ is included in or equal to another XML language $M$.
- It is also decidable if a regular language $L \subseteq D_A$ is an XML language.
- It is however undecidable if a context-free language is an XML language.
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Motivation

Why should we care about one-unambiguous regular languages?

Because in SGML the regular expressions (more precisely: model groups) on the right-hand side of productions have to be one-unambiguous.

But who cares about SGML?

The W3C does in its recommendation for XML:

*For compatibility [with SGML], it is an error if the content model allows an element to match more than one occurrence of an element type in the content model.*

Furthermore one-unambiguity helps to efficiently parse a document.
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Description of (One-)Unambiguous Regular Expressions

Informal Description

- If we can determine uniquely which symbol of a regular expression corresponds to a symbol in the input word (while knowing the whole word), the regular expression is **unambiguous**.
- If we can do so without looking beyond that symbol, the regular expression is **one-unambiguous**.

Example

- \((bc) + (bd)\) is unambiguous, but not one-unambiguous,
- \(b(c + d)\) is one-unambiguous (hence also unambiguous).

A more formal way to distinguish between symbols is needed ⇒ marking (soon).
Description of (One-)Unambiguous Regular Expressions

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- If we can determine uniquely which symbol of a regular expression corresponds to a symbol in the input word (while knowing the whole word), the regular expression is unambiguous.
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A New Perspective on the First Section

How is one-unambiguity incorporated into the XML grammars and languages of the previous section?

It is not. Thus the XML languages of the previous section are not even proper DTD languages.

Example: an XML language lacking one-unambiguity

\[
\begin{align*}
N &= \{X_a, X_b\} \\
T &= \{a, \overline{a}, b, \overline{b}\} \\
S &= X_a \\
P &= \{X_a \rightarrow aX_b^*X_b^*\overline{a}, \\
& \quad X_b \rightarrow b \text{ something } \overline{b}\}
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\[ P = \{ X_a \rightarrow aX_b^*X_b^*\overline{a}, \]
\[ X_b \rightarrow b \text{ something } \overline{b} \} \]
Example: \((a + b)^*a(ab)^*\)

- \((a_1 + b_1)^*a_2(a_3b_2)^*\) is a marking,
- \((a_4 + b_2)^*a_1(a_5b_1)^*\) is a marking,
- \((a_1 + b_2)^*a_3(a_1b_1)^*\) is not a marking.

Definition

- Assigning subscripts to occurrences of symbols,
- subscript is unique for each sort of symbols,
- marking of a regular expression \(E\) over alphabet \(\Sigma\) denoted by \(E'\) over the alphabet \(\Pi\),
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- Assigning subscripts to occurrences of symbols,
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Definition of One-Unambiguous Regular Languages

**Definition**

Let $t, u, v, w$ be words over $\Pi$ and $x, y \in \Pi$. A regular expr. $E$ is one-unambiguous iff

$$uxv, uyw \in L(E') \land x \neq y \Rightarrow x^{|} \neq y^{|}.$$ 

If $\exists$ one-unambiguous $E$ for $L \Rightarrow L$ is one-unambiguous.

**Examples**

- $E = (bc) + (bd)', E' = (b_1c_1) + (b_2d_1), b_1c_1 \in L(E'), b_2d_1 \in L(E')$: $b_1 \neq b_2$, but $b = b$ therefore $E$ is not one-unambiguous.

- $F = b(c + d), F' = b_1(c_1 + d_1)$ satisfies the conditions $\Rightarrow F$ is one-unambiguous.
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Definition of first, last and follow

**Definition**

Let $L$ be a language.

- $\text{first}(L) := \{ b \mid \text{there is a word } w \text{ such that } bw \in L \}$
- $\text{last}(L) := \{ b \mid \text{there is a word } w \text{ such that } wb \in L \}$
- $\text{follow}(L, a) := \{ b \mid \text{there are words } v \text{ and } w \text{ such that } vabw \in L \}$, for each symbol $a$

For a regular expression $E$ we define $\text{set}(E)$ as $\text{set}(L(E))$.

**Example:** $E = b(c + d)$

- $\text{first}(E) = \{ b \}$, $\text{last}(E) = \text{follow}(E, b) = \{ c, d \}$,
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An Alternative Definition of One-Unambiguity

Theorem

A regular expression $E$ is one-unambiguous iff

1. $\forall x, y \in \text{first}(E') : x \neq y \Rightarrow x^\|$ $\neq$ $y^\|$, 
2. $\forall z \in \text{sym}(E') \land x, y \in \text{follow}(E', z) : x \neq y \Rightarrow x^\| \neq y^\|$, 

where $\text{sym}(E')$ is the set of symbols occurring in $E'$.

Example: $E = b(c + d)$ marked as $b_1(c_1 + d_1)$

- $\text{first}(E') = \{b_1\}$ (condition 1 is satisfied),
- $\text{follow}(E, c_1) = \text{follow}(E, d_1) = \emptyset$, $\text{follow}(E, b) = \{c_1, d_1\}$; $c_1 \neq d_1 \Rightarrow c \neq d$ (condition 2 is satisfied).

$E$ is one-unambiguous.
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Definition

Let $E$ be a regular expression. The corresponding Glushkov automaton $G_E = (Q_E, \Sigma, \delta_E, q_I, F_E)$ is defined by:

1. $Q_E := \text{all symbols of } E' \text{ and a new, initial state } q_I$,
2. for $a \in \Sigma$: $\delta_E(q_I, a) := \{ x \mid x \in \text{first}(E'), x \|^a = a \}$,
3. for $x \in \text{sym}(E')$ and $a \in \Sigma$: $\delta_E(x, a) = \{ y \mid y \in \text{follow}(E', x), y \|^a = a \}$,
4. $F_E = \left\{ \begin{array}{ll}
\text{last}(E') \cup \{ q_I \}, & \text{if } \varepsilon \in L(E) \\
\text{last}(E'), & \text{otherwise}.
\end{array} \right.$
Glushkov Automata 1/4

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3. for $x \in \text{sym}(E')$ and $a \in \Sigma$: $\delta_E(x, a) = \{ y \mid y \in \text{follow}(E', x), y^\dagger = a \},$
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Example: \((a + b)^*a + \varepsilon\) marked as \((a_1 + b_1)^*a_2 + \varepsilon\)
Example: $a^* ba^*$ marked as $a_1^* b_1 a_2^*$
Glushkov Automata 4/4

- No transition leads back to the initial state.
- Two transitions that lead to the same state have identical labels.
- $G_E$ can be computed in time quadratic in the size of $E$.

**Theorem**

A regular expression $E$ is one-unambiguous iff $G_E$ is a DFA.

With Glushkov automata we can decide rather efficiently if a regular expression is one-unambiguous.
Glushkov Automata 4/4

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Overview

1. XML Languages and Grammars
   - Introduction and Basics
   - Characterization

2. One-Unambiguous Regular Languages
   - Introduction and Basics
   - Recognition
   - Closure

3. Analysis of XML Schema Languages
   - Introduction and Basics
   - Language Classes
   - Evaluating XML Schema Languages
We know (mostly from the GTI lecture) . . .

- . . . that for each regular language $L$ the corresponding minimum-state DFA $MS(L)$ is uniquely determined.
- . . . how minimizing a DFA can be achieved by equivalence-class construction.
- . . . that we can transform an NFA to an equivalent DFA using subset construction.
- . . . how to transform a regular expression to a Glushkov automaton.
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Idea: Examine the structural properties of $MS(L)$ that characterize an one-unambiguous language $L$.

If $E$ is a regular expression, $MS(L(E))$ can be achieved by minimizing $G_E$.

If $E$ is one-unambiguous, we do not need to use subset construction on $G_E$, because $G_E$ already is a DFA.

Question: What properties of Glushkov automata are preserved under minimization, but not necessarily under subset construction?
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**Initial Considerations 2/2**

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- **Question**: What properties of Glushkov automata are preserved under minimization, but not necessarily under subset construction?
**Definition: Orbit**

For $q$ being a state of an NFA, $O(q)$ is the strongly connected component of $q$.

**Example**

- $O(q_1) = \{q_1\}$
- $O(q_3) = \{q_2, q_3, q_4\}$
- $O(q_2) = \{q_2, q_3, q_4\}$
- $O(q_4) = \{q_2, q_3, q_4\}$
**Gates**

**Definition**

If \( q \in F \) or there exists \( q' \notin O(q) : ((q, a), q') \in \delta \), then \( q \) is a gate of \( O(q) \).

**Example**

- \( q_1 \) and \( q_2 \) are not gates of their orbits.
- \( q_3 \) and \( q_4 \) are gates of their orbits.
Definition

An NFA has the **orbit property** if all gates of each orbit have identical connections to the outside world.

Example

- The left diagram represents an NFA with the orbit property.
- The right diagram represents an NFA without the orbit property.

The example diagrams illustrate the concept with states labeled $q_1$, $q_2$, $q_3$, and $q_4$, and transitions labeled $a$, $b$, $c$, and $d$.
Definition: Orbit Automaton

1. For a state $q$, restrict state set to $O(q)$,
2. set $q$ as the initial state,
3. set the gates of $O(q)$ as the final states,
4. denote the resulting automaton as $M_q$.

Definition

- The language of $M_q$ is called the orbit language of $q$.
- The languages $L(M_q), q \in Q_M$ are called the orbit languages of $M$. 
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Orbit Automata and Orbit Languages 1/2

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**Stefan Tittel**

Formal Language Foundations and Schema Languages
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Orbit Automata and Orbit Languages 2/2

Example

\[
\begin{array}{c}
M_{q_1} \\
M_{q_3} \\
M_{q_2}
\end{array}
\]

\[
\begin{array}{c}
q_1 \\
q_2 \\
q_3 \\
q_4
\end{array}
\]
Characterization of One-Unambiguous Regular Languages

Theorem

M is a minimal DFA. If and only if

- M has the orbit property,
- all orbit languages of M are one-unambiguous,

then \( L(M) \) is one-unambiguous.

An one-unambiguous regular expression for \( L(M) \) is constructable from the one-unambiguous regular expressions for the orbit languages.

Definition

\( \mathcal{O}(q) \) is trivial if \( \mathcal{O}(q) = \{q\} \) and \( (q, q) \notin \delta \).

Question: How can we decide if an orbit language is one-unambiguous if the orbit is not trivial?
Characterization of One-Unambiguous Regular Languages

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Characterization of One-Unambiguous Regular Languages

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**M-Consistency**

**Definition**

- $M$ is a DFA,
- $s \in \Sigma_M$ is $M$-consistent if
  \[ \exists f(s) \in Q_M : \forall q \in F_M : ((q, s), f(s)) \in \delta_M, \]
- $S \subseteq \Sigma_M$ is $M$-consistent if $\forall s \in S : s$ is $M$-consistent.

**Example**

- $a$ is $M_1$-consistent
- $a$ is not $M_2$-consistent
**S-Cut**

**Definition: S-Cut** $M_S$ of $M$

\[
\forall a \in S : \forall q \in Q_M : \forall q' \in F_M : \text{remove } ((q, a), q') \text{ from } \delta_M
\]

**Example**

- Original Automaton $M$
- $\{a, b\}$-cut of $M$
Theorem

Let

- $M$ be a minimal DFA,
- $S$ be an $M$-consistent set of symbols,

then iff

- $M_S$ satisfies the orbit property,
- all orbit languages of $M_S$ are one-unambiguous,

then $L(M)$ is one-unambiguous.

We will extend this theorem to a decision algorithm very soon.
Conditions for a DFA to Be One-Unambiguous 2/2

Example

The \(\{a, b\}\)-cut of \(M\) has only one-unambiguous orbits. Hence \(L(M)\) is one-unambiguous and can be denoted by the one-unambiguous regular expression \(c(a + b(\epsilon + cc))^*\).
boolean one-unambiguous (MinimalDFA M) {
    compute \( S := \{ a \in \Sigma \mid a \text{ is } M\text{-consistent}\} \);
    if (\( M \) has a single, trivial orbit) \{return true;\}
    if (\( M \) has a single, nontrivial orbit && \( S = \emptyset \)) \{return false;\}
    compute the orbits of \( M_S \);
    if (\! OrbitProperty(\( M_S \))) \{return false;\}
    for (each orbit \( K \) of \( M_S \)) {
        choose \( x \in K \);
        if (\! one-unambiguous((\( M_S \))_x)) \{return false;\}
    }
    return true;
}
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Closure

Definition

- $L$ is a language,
- $w$ is a word,
- $\{v \mid vw \in L\}$ is the derivative of $L$ with respect to $w$ and denoted by $w\setminus L$.

- The family of one-unambiguous regular languages is closed under derivatives.
- One-unambiguous regular expressions are not closed under derivatives, unless they are in a star normal form.
- The family of one-unambiguous regular languages is not closed under union, concatenation or star.
Closure

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- $L$ is a language,
- $w$ is a word,
- $\{v \mid wv \in L\}$ is the derivative of $L$ with respect to $w$ and denoted by $w \backslash L$.

- The family of one-unambiguous regular languages is closed under derivatives.
- One-unambiguous regular expressions are not closed under derivatives, unless they are in a star normal form.
- The family of one-unambiguous regular languages is not closed under union, concatenation or star.
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Overview

1. XML Languages and Grammars
   - Introduction and Basics
   - Characterization

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An XML schema describes constraints on the structure and content beyond the basic syntax constraints of XML itself. It is specified by an XML schema language.

Examples of XML schema languages:
- DTD
- XML Schema
- RELAX (NG)
- DSD
- XDuce

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“XML schema” ≠ “XML Schema”
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Attention

“XML schema” ≠ “XML Schema”
Example: XML Schema Specification of a Business Card (Extract)

```xml
<schema [...]>
  <element name="card" type="b:card_type"/>
  <element name="name" type="string"/>
  <element name="logo" type="b:logo_type"/>
  <complexType name="card_type">
    <sequence>
      <element ref="b:name"/>
      <element ref="b:logo" minOccurs="0"/>
    </sequence>
  </complexType>
  <complexType name="logo_type">
    <attribute name="url" type="anyURI"/>
  </complexType>
</schema>
...
Motivation

We are interested in . . .

1. . . . expression power.
2. . . . closure properties.
3. . . . document validation.

Examples

1. Can I model my constraints with a certain XML schema language?
2. What XHTML 1.0 documents are still valid XHTML 1.1 documents?
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A **model group** is a regular expression in which the following additional operators are allowed:

- \( ? \) – where \( E? \) denotes \( L(E + \epsilon) \)
- \( \& \) – where \( F \& G \) denotes \( L(FG + GF) \)
- \( + \) – where \( E^+ \) denotes \( L(EE^*) \)

**Definition: Regular Tree Grammar** \( G = (N, T, P, S) \)

- \( N \) = non-terminal symbols,
- \( T \) = terminal symbols,
- \( P \) = productions of the form \( X \rightarrow a \ Expression \) with \( X \in N \), \( a \in T \) and \( \Expression \) model group over \( N \),
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Example: A Tree Grammar for a DTD

```xml
<!DOCTYPE book [ 
  <!ELEMENT book (author+, publisher) >
  <!ELEMENT author (#PCDATA) >
  <!ELEMENT publisher (EMPTY) >
  <!ATTLIST publisher Name CDATA #IMPLIED >
]>
```

\[
\begin{align*}
  N &= \{ \text{Book, Author, Publisher, Pcdata} \}, \\
  T &= \{ \text{book, author, publisher, pcdata} \}, \\
  S &= \{ \text{Book} \}, \\
  P &= \{ \text{Book} \rightarrow \text{book}(\text{Author}^+, \text{Publisher}), \\
      \text{Author} \rightarrow \text{author}(\text{Pcdata}), \\
      \text{Publisher} \rightarrow \text{publisher}(\varepsilon), \\
      \text{Pcdata} \rightarrow \text{pcdata}(\varepsilon) \}. \\
\end{align*}
\]
Example

A possible document complying with this DTD:

```xml
<book>
  <author>J. E. Hopcroft</author>
  <author>J. D. Ullman</author>
  <publisher Name="Addison-Wesley"/>
</book>
```

An instance tree for this document:
Normal Form 1 (NF1) 1/2

Definition: Grammar in Normal Form 1 (NF1)

Grammar \( G = (N1, N2, T, P1, P2, S) \) with

- \( T \) and \( S \) as usual,
- \( N1 \) = non-terminal symbols used for deriving trees,
- \( N2 \) = non-terminal symbols used for content-model spec.,
- \( P1 \) = productions of the form \( A \rightarrow aX \) with \( A \in N1, X \in N2, a \in T \) (only one production per symbol \( \in N1 \)),
- \( P2 \) = prod. of the form \( X \rightarrow \text{Exp} \) with \( X \in N2, \text{Exp} \) model group over \( N1 \) (only one production per symbol \( \in N2 \)).

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\( \text{contentModel}(A) \ (A \in N1) \) is the model group over \( N1 \) denoting the content of \( A \).
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Stefan Tittel

Formal Language Foundations and Schema Languages
Example: The Grammar of the Last Example in NF1

\[
N_1 = \{\text{Book, Author, Publisher, Pcdata}\},
\]
\[
N_2 = \{\text{BOOK, AUTHOR, PUBLISHER, PCDATA}\},
\]
\[
T = \{\text{book, author, publisher, pcdata}\},
\]
\[
P_1 = \{\text{Book} \rightarrow \text{book BOOK, Author} \rightarrow \text{author AUTHOR, Publisher} \rightarrow \text{publisher PUBLISHER, Pcdata} \rightarrow \text{pcdata PCDATA}\},
\]
\[
P_2 = \{\text{BOOK} \rightarrow (\text{Author}^+, \text{Publisher}), \text{AUTHOR} \rightarrow \text{Pcdta, PUBLISHER} \rightarrow \varepsilon, \text{PCDATA} \rightarrow \varepsilon\},
\]
\[
S = \{\text{Book}\}.
\]

\[\text{contentModel(Book)} = (\text{Author}^+, \text{Publisher})\]

From now on upper- and lower-casing will be used like in this example to distinguish between symbols in $N_1$, $N_2$ and $T$. 
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& \phantom{=} \text{Publisher} \rightarrow \text{publisher PUBLISHER, Pcdata} \rightarrow \\
& \phantom{=} \text{pcdata PCDATA} \}, \\
P2 &= \{ \text{BOOK} \rightarrow (\text{Author}^+, \text{Publisher}), \text{AUTHOR} \rightarrow \text{Pcdata}, \\
& \phantom{=} \text{PUBLISHER} \rightarrow \varepsilon, \text{PCDATA} \rightarrow \varepsilon \}, \\
S &= \{ \text{Book} \}.
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Formal Language Foundations and Schema Languages
Local Tree Grammars

Definition: Tree-Locality Constraint

∀a ∈ T there is no more than one rule of the form A → aX in P1.

Definition: Local Tree Grammar (LTG)

A regular tree grammar that satisfies the tree-locality constraint.

Example

\[
\begin{align*}
N_1 &= \{ \text{Out, In, Pcd} \} \\
N_2 &= \{ \text{OUT, IN, PCD} \} \\
T &= \{ \text{out, in, pcd} \} \\
P_{1a} &= \{ \text{Out} \rightarrow \text{out OUT}, \text{In} \rightarrow \text{in IN}, \text{Pcd} \rightarrow \text{pcd PCD} \} \\
P_{1b} &= \{ \text{Out} \rightarrow \text{out OUT}, \text{In} \rightarrow \text{out IN}, \text{Pcd} \rightarrow \text{pcd PCD} \} \\
P_2 &= \{ \text{OUT} \rightarrow \text{In, IN} \rightarrow \text{Pcd}, \text{PCD} \rightarrow \varepsilon \} \\
(N_1, N_2, T, P_{1a}, P_2) &\text{ is an LTG, } (N_1, N_2, T, P_{1b}, P_2) \text{ is not.}
\end{align*}
\]
Local Tree Grammars

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\((N_1, N_2, T, P_{1a}, P_2)\) is an LTG, \((N_1, N_2, T, P_{1b}, P_2)\) is not.
Definition

Two different non-terminals $A$ and $B$ are called competing with each other if

- one production rule has $A$ in the left-hand side,
- another production rule has $B$ in the left-hand side, and
- these two production rules share the same terminal in the right-hand side.

Definition: Single-Type Constraint Grammar

- For each production rule, non-terminals in its content model do not compete with each other,
- start symbols do not compete with each other.
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**Definition**

A tree language is a **single-type constraint language** if it is generated by a single-type constraint grammar.

**Example**

- $P_1 = \{A \rightarrow B, A \rightarrow C, B \rightarrow a, C \rightarrow b\}$ satisfies the s.-t. c.,
- $P_2 = \{A \rightarrow B, A \rightarrow C, B \rightarrow a, C \rightarrow a\}$ doesn’t.

Single-type constraint languages and local tree languages are . . .

- . . . closed under intersection.
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Local Tree Languages $\subset$ Single Type Constraint Languages

**Theorem**

*Local tree languages form a proper subclass of single-type constraint languages.*

**Proof:**

$\Rightarrow$: A local tree language satisfies the single-type constraint by definition.

$\Leftarrow$:

- Consider a regular tree grammar with $A, B \in N1 \land A \neq B \land \text{root}(A) = \text{root}(B)$.
- This grammar can satisfy the single-type constraint.
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**DTD**
- TDLL(1),
- local tree grammar.

**DSD**
- No constraints on the production rules,
- theoretically any regular tree grammar can be expressed in DSD,
- parsing algorithm uses greedy technique with one vertical and horizontal lookahead,
- acceptance of all and only TDLL(1) languages is suspected.
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XML Schema and RELAX

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Expression Power

This figure is from [3].

(a) regular tree grammars (*RELAX*, *XDuce*)
(b) TD(1) grammars
(c) single-type constraint grammars
(d) local tree grammars
(e) TDLL(1) grammars
(f) TDLL(1) w/ single-type constraint (*XML Schema*, DSD?)
(g) TDLL(1) w/ tree-locality constraint (*DTD*)


